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Chaos synchronization of general Lorenz, Lü, and Chen systems

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Abstract

Chaos synchronization of general Lorenz, Lü, and Chen chaotic systems by using active control and nonlinear control is studied in this paper. Active control method based on Hurwitz stability criterion with pole-placement scheme and nonlinear control based on Lyapunov stability theory are introduced to design controller to synchronize two identical chaotic systems. We demonstrate that a coupled general Lorenz, Lü, and Chen chaotic systems can be synchronized. Numerical simulations are used to show the effectiveness of the proposed control method.

Keywords: Chaos synchronization, Active control, Nonlinear control.

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廣義勞倫茲系統之渾沌同步

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摘 要

本文是利用主動控制與非線控制兩種方法對廣義勞倫茲(Lorenz, Lü, and Chen)渾沌系統作渾沌同步控制。主動控制方法是依據 Hurwitz 穩定法則及極點安置規劃來設計控制器，而非線控制方法是依據李雅普若夫(Lyapunov)穩定法則來設計控制器。使用這兩種控制方法，我們可以設計渾沌控制器，使得兩個渾沌系統可以做同步運動。另外我們也將作數值模來驗證控制器的效果。

關鍵詞：渾沌同步、主動控制、非線控制。

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1.Introduction

Since Pecora and Carroll introduced a method [1] to synchronize two identical chaotic systems with different initial conditions, chaos synchronization, as a very important topic in nonlinear science, has been developed extensively in the last few years [2-9]. Many scientists have been attracted to investigate chaos synchronization in various fields including secure communications, optics, chemical and biological systems, etc. During the last decades, many methods have been successfully applied to chaos synchronization such as linear feedback control [2], adaptive control [3-4], backstepping design [5], active control [6-7], and nonlinear control [8-9], etc.

The aim of this paper is to apply active control and nonlinear control to synchronize two identical general Lorenz, Lü, and Chen chaotic systems. This paper organized as follows: In Section 2 the controlled system model is described. In Section 3 we apply active control to achieve chaos synchronization of two identical chaotic systems and numerical simulations are used to show this process. In Section 4 we apply nonlinear control to achieve chaos synchronization of two identical chaotic systems and numerical experiments are used to show such synchronization. Finally, the conclusions of this paper are briefly stated.

2.Design of controller

Consider a chaotic system in the form of

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + f(\mathbf{x}) \tag{1}$$

where $\mathbf{x} \in R^n$ is the state vector, $\mathbf{A} \in R^{n\times n}$ are matrices of system parameters, and $f(\mathbf{x})$ is a continuous nonlinear function. Eq. (1) is considered as a drive system. The corresponding controlled response system is given by

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + f(\mathbf{y}) + \mathbf{u} \tag{2}$$

where $\mathbf{y} \in R^n$ denotes the state vector of the response system and $\mathbf{u}$ is a nonlinear stabilizing state feedback controller to be designed later.

From (1)-(2), the error dynamics can be obtained in the form of

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + h(\mathbf{x}, \mathbf{e}) + \mathbf{u}\,, \tag{3}$$

where $\mathbf{e} = \mathbf{y} - \mathbf{x}$ is the state error vector, $h(\mathbf{x}, \mathbf{e}) = f(\mathbf{x} + \mathbf{e}) - f(\mathbf{x})$ and $h(\mathbf{x}, 0) = 0$. The aim of synchronization is to make $\lim_{t\to\infty} \| \mathbf{e}(t) \| = 0$. In this paper, we introduce active control and nonlinear control methods based on the exact linearization approach to design controller to synchronize two identical chaotic systems.

In active control method, we can choose

$\mathbf{u} = -h(\mathbf{x}, \mathbf{e}) + \mathbf{v}$ to cancel the nonlinear function $h(\mathbf{x}, \mathbf{e})$. This cancellation results in a linear system

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{v}. \tag{4}$$

Thus, the stabilization problem for the nonlinear system has been reduced to a stabilization problem for a controllable linear system. Now apply the linear state feedback control $\mathbf{v} = -\mathbf{K}\mathbf{e}$ and design a matrix $\mathbf{K}$ to assign the eigenvalues of (**A**-**K**) to desired locations in the open left-half complex plane, i.e., (**A**-**K**) is Hurwitz. Then the states of response system and drive system are synchronized asymptotically globally.

In nonlinear control method, we can choose $\mathbf{u} = -h(\mathbf{x}, \mathbf{e}) + \mathbf{w}$ to cancel the nonlinear function $h(\mathbf{x}, \mathbf{e})$. This cancellation also results in a linear system as Eq. (4). Based on Lyapunov stability theory, when controller **w** makes that Lyapunov error function $V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^T\mathbf{e}$ is a positive function and its derivative of $V(\mathbf{e})$ is a negative function, the synchronization of two identical general Lorenz, Lü, and Chen chaotic systems from differential initial conditions is achieved.

3.Synchronization of two identical chaotic systems

3.1 Design of controller via active control method

Consider the unified chaotic systems described by

$$\begin{aligned}
\dot{x}_1 &= (25\theta + 10)(y_1 - x_1), \\
\dot{y}_1 &= (28 - 35\theta)x_1 - x_1 z_1 + (29\theta - 1)y_1, \\
\dot{z}_1 &= x_1 y_1 - ((8 + \theta)/3)z_1,
\end{aligned} \tag{5}$$

where $\theta \in [0, 1]$. As θ changes continuously from 0 to 1, the system remains continuously to be chaotic. System (5) is considered as a drive system called the general Lorenz, Lü, and Chen system as $\theta \in [0, 0.8)$, $\theta = 0.8$, $\theta \in (0.8, 1]$, respectively.

The corresponding controlled response system is given by

$$\begin{aligned}
\dot{x}_2 &= (25\theta + 10)(y_2 - x_2) + u_1, \\
\dot{y}_2 &= (28 - 35\theta)x_2 - x_2 z_2 + (29\theta - 1)y_2 + u_2, \\
\dot{z}_2 &= x_2 y_2 - ((8 + \theta)/3)z_2 + u_3,
\end{aligned} \tag{6}$$

where u_1, u_2 and u_3 are nonlinear stabilizing state feedback controller. Subtracting Eqs. (5) from (6), we get the following error dynamical system:

$$\begin{aligned}
\dot{e}_1 &= (25\theta + 10)(e_2 - e_1) + u_1, \\
\dot{e}_2 &= (28 - 35\theta)e_1 - x_2 z_2 + x_1 z_1 + (29\theta - 1)e_2 + u_2 \\
\dot{e}_3 &= x_2 y_2 - x_1 y_1 - ((8 + \theta)/3)e_3 + u_3,
\end{aligned} \tag{7}$$

where $e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1$ are the tracking error states.

In this study, we rewrite system (7) in the form

$$\dot{\mathbf{e}} = \mathbf{Ae} + h(x, \mathbf{e}) + \mathbf{u} \quad (8)$$

where $\mathbf{e} = [e_1, e_2, e_3]^T$, $\mathbf{u} = -h(\mathbf{x}, \mathbf{e}) + \mathbf{v}$, and

$$\mathbf{A} = \begin{bmatrix} -(25\theta+10) & (25\theta+10) & 0 \\ (28-35\theta) & (29\theta-1) & 0 \\ 0 & 0 & -(8+\theta)/3 \end{bmatrix},$$

$$h(\mathbf{x}, \mathbf{e}) = \begin{bmatrix} 0 \\ -z_1e_1 - x_1e_3 - e_1e_3 \\ y_1e_1 + x_1e_2 + e_1e_2 \end{bmatrix}.$$

According to the feedback linearization approach, we choose $\mathbf{u} = -h(\mathbf{x}, \mathbf{e}) + \mathbf{v}$ to cancel the nonlinear function $h(\mathbf{x}, \mathbf{e})$ and to impose a desired linear dynamics. This cancellation results in a linear system

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{v}\,. \quad (9)$$

Assuming that the parameters of the drive and response systems are known and the states of both systems are measurable. Thus, the stabilization problem for the nonlinear system has been reduced to a stabilization problem for a controllable linear system. Now apply the linear state feedback control $\mathbf{v} = -\mathbf{Ke}$ and design a matrix **K** to assign the eigenvalues of (**A**-**K**) to desired locations in the open left-half complex plane, i.e., (**A**-**K**) is Hurwitz. Then the states of response system and drive system are synchronized asymptotically globally.

There are many possible choices for the control **v**. Let us design **K** to assign the eigenvalues of (**A**-**K**) at –1, -1, and –1. The gain matrix **K** is given by

$$\mathbf{K} = \begin{bmatrix} -(25\theta+10)+1 & (25\theta+10) & 0 \\ (28-35\theta) & (29\theta-1)+1 & 0 \\ 0 & 0 & -(8+\theta)/3+1 \end{bmatrix}.$$

3.2 Design of controller via nonlinear control method

In this case study, the error dynamics of the system is also in the same form as Eq. (8). we choose $\mathbf{u} = -h(\mathbf{x}, \mathbf{e}) + \mathbf{w}$ to cancel the nonlinear function $h(\mathbf{x}, \mathbf{e})$ and to impose a linear dynamics.

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{w}\,. \quad (10)$$

where $\mathbf{w} = [w_1, w_2, w_3]^T$, and

$$\mathbf{A} = \begin{bmatrix} -(25\theta+10) & (25\theta+10) & 0 \\ (28-35\theta) & (29\theta-1) & 0 \\ 0 & 0 & -(8+\theta)/3 \end{bmatrix}.$$

Take a Lyapunov function candidate as

$$V(\mathbf{e}) = \tfrac{1}{2}\mathbf{e}^T\mathbf{e}\,. \quad (11)$$

We get the time derivative of the Lyapunov function (11) is

$$
\begin{aligned}
\dot{V} &= e_1[(25\theta+10)(e_2-e_1)+w_1]+e_2[(28-35\theta)e_1 \\
&\quad +(29\theta-1)e_2+w_2]+e_3[-((8+\theta)/3)e_3+w_3] \\
&= -(25\theta+10)e_1^2+(38-10\theta)e_1e_2+(29\theta-1)e_2^2 \\
&\quad -((8+\theta)/3)e_3^2+e_1w_1+e_2w_2+e_3w_3
\end{aligned}
\tag{12}
$$

Select

$w_1=-(38-10\theta)e_2$, $w_2=-29\theta e_2$, $w_3=0$.

Then

$$
\dot{V}=-(25\theta+10)e_1^2-e_2^2-(8+\theta)e_3^2/3)\le -2V(\mathbf{e}). \tag{13}
$$

Therefore, $\frac{1}{2}\|\mathbf{e}(t)\|^2\le V(\mathbf{e})=\frac{1}{2}\mathbf{e}^T\mathbf{e}\le V(0)e^{-2t}$, implying that synchronization of the derive response systems is achieved and the state error **e** converges to the origin with a rate of at least -1.

4.Numerical results

The numerical simulations are carried out as shown in Fig. 1-5. In these numerical simulations, the fourth-order Runge-Kutta method is used to solve the system. The initial values of the drive system and response system are taken as

$x_1(0)=1,\ y_1(0)=1,\ z_1(0)=1$,

$x_2(0)=-10,\ y_2(0)=-17,\ z_2(0)=15$,

respectively. When $\theta=0, 0.8, 1$, the system (5) has Lorenz, Lü, and Chen chaotic attractors as shown in Fig. (1)-(3), respectively. Fig. (4) shows the synchronization error of two identical systems at $\theta=1$ will converge to zero after applying active control at $t=25$. Fig. (5) shows the synchronization error of two identical systems at $\theta=1$ will converge to zero after applying nonlinear control at $t=10$. So we can see that two identical chaotic systems from different initial values are indeed achieving chaos synchronization by using active control and nonlinear control.

5.Conclusion

This paper demonstrates that chaos in general Lorenz, Lü, and Chen systems can be controlled using active control and nonlinear control. The simulation results show that the states of two identical systems are synchronized asymptotically globally.

Acknowledgments

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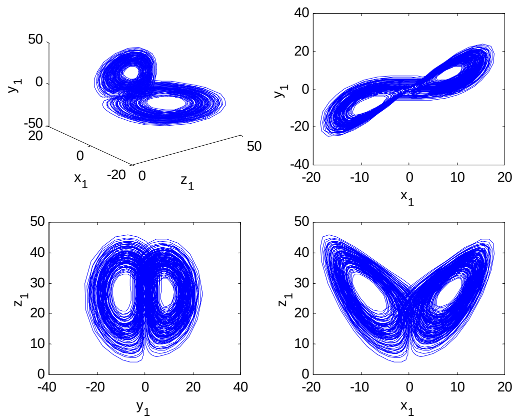

Fig. 1. The Lorenz attractor of the chaotic system (5) at $\theta = 0$.

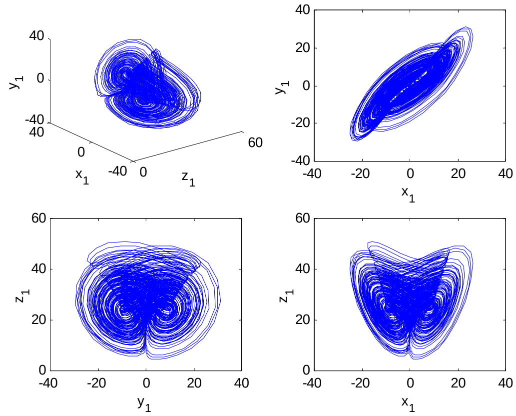

Fig. 2. The Lü attractor of the chaotic system (5) at $\theta = 0.8$.

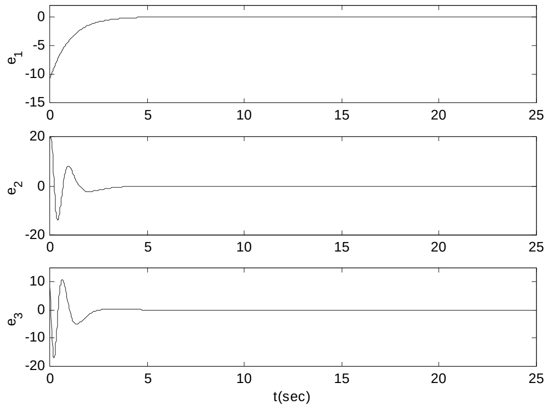

Fig. 4. Synchronization errors of two identicalchaotic systems (8) at $\theta = 1$.

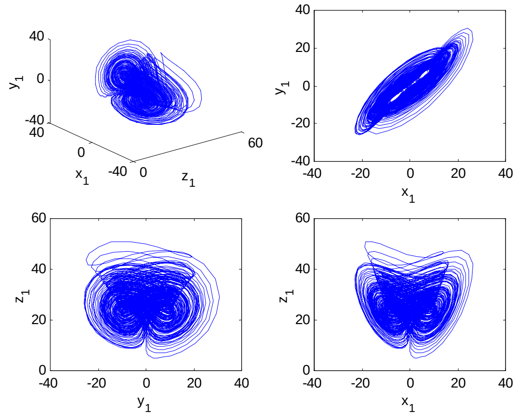

Fig. 3. The Chen attractor of the chaotic system (5) at $\theta = 1$.

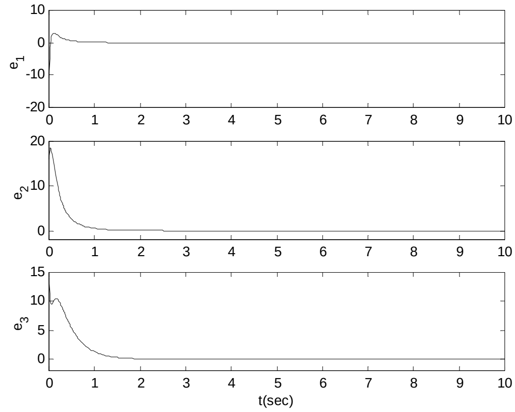

Fig. 5. Synchronization errors of two identical chaotic systems (10) at $\theta = 1$.